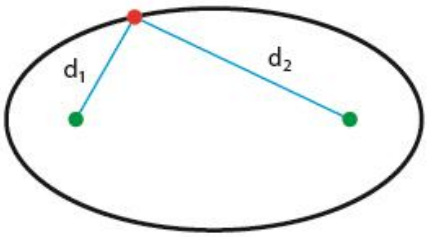
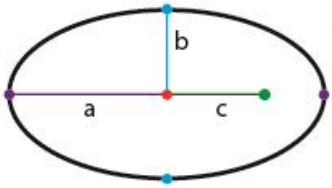
Conic Sections: Ellipses

An ellipse is the set of all points where the sum of the distance from 2 given points (foci) and the curve is constant.

ie, $d_1 + d_2 =$ a constant

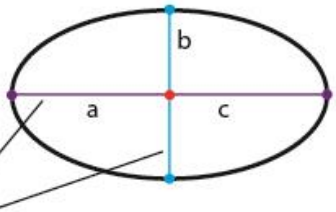


If the ellipse is horizontal: The equation is: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$



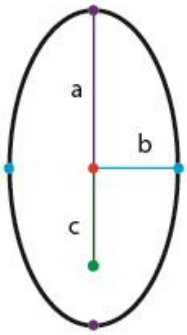
$c^2 = a^2 - b^2$

- -- the center = (h, k)
- -- a focus (there are 2) = $(h \pm c, k)$
- -- vertices
- --covertices



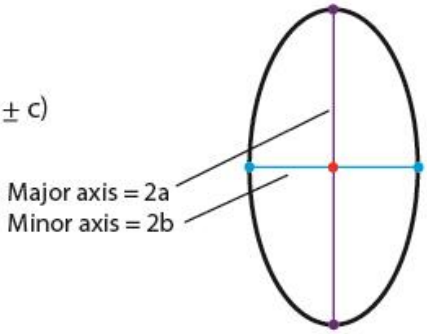
Major axis = 2a
Minor axis = 2b

If the ellipse is vertical: The equation is: $\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$



$c^2 = a^2 - b^2$

- -- the center = (h, k)
- -- a focus (there are 2) = $(h, k \pm c)$
- -- vertices
- --covertices



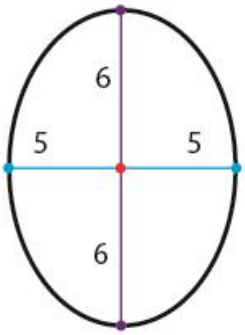
Ex: $\frac{(x-2)^2}{25} + \frac{(y+3)^2}{36} = 1$

The center is (2, -3)
$a = \sqrt{25} = 5$
$b = \sqrt{36} = 6$ b > a so the ellipse is vertical
$c^2 = b^2 - a^2 = 36 - 25$, so $c = \sqrt{11}$

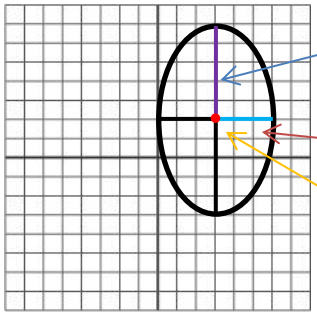
Foci are $(2, -3 + \sqrt{11})$ and $(2, -3 - \sqrt{11})$

Major axis $= 2 \cdot 6 = 12$

Minor axis $= 2 \cdot 5 = 10$



Ex:



$a = 4$, $a^2 = 16$

$b = 3$, $b^2 = 9$

(h, k) = (3, 2)

Write the equation of the ellipse.

$\frac{(x+h)^2}{b^2} + \frac{(y+k)^2}{a^2} = 1$ so $\frac{(x+3)^2}{9} + \frac{(y+2)^2}{16} = 1$

Ex: For the ellipse: $\frac{(x+2)^2}{49} + \frac{(y-3)^2}{16} = 1$, what are the coordinates of the foci? h = –2, k = 3,

$a^2 = 49$, $b^2 = 16 \rightarrow c^2 = a^2 - b^2 = 49 - 16 = 33 \rightarrow c = \sqrt{33} \rightarrow$ foci = $(-2 \pm \sqrt{33}, 3)$